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Self-focusing magnetostatic beams in thin magnetic films

R. Khomeriki[a]

Department of Physics, Tbilisi State University, Chavchavadze ave. 3, Tbilisi 0128, Republic of Georgia

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Abstract. The possibility of generation of stable self-focusing beams in in-plane magnetized thin magnetic films is considered, and theoretical conditions for the existence of such localized solutions are discussed. It is shown that for the definite direction between static magnetizing field and preferential direction of radiation from a microwave antenna, the problem reduces to the one-dimensional nonlinear Schrödinger equation. For such angles it is possible to generate stable self-focusing beams. Particular values of beam width and propagation angles versus magnitude of magnetizing field are calculated in order to suggest a realistic experimental setup for the observing the discovered effect.

PACS. 85.70.Ge Ferrite and garnet devices – 75.30.Ds Spin waves – 76.50.+g Ferromagnetic, antiferromagnetic, and ferrimagnetic resonances; spin-wave resonance

1 Introduction

Observations of magnetostatic solitons in thin magnetic films together with experiments in nonlinear optics are major testing grounds for recent advances in nonlinear physics. In full accordance with theoretical predictions [1], magnetostatic bright envelope solitons have been observed in both in-plane [2–5] and perpendicularly magnetized [6–8] quasi-one-dimensional yttrium-iron garnet thin films (magnetic waveguides). On the other hand, as expected, dark surface wave magnetostatic solitons have been observed only in in-plane magnetized films [9–11]. Moreover, in full analogy with light bullets in nonlinear optics [12,13], spin-wave metastable bullets have been found in wide magnetic films [14,15]. The only difference between nonlinear processes in magnetic films and optical devices is that self-focusing magnetostatic beams are unstable at relatively long distances, unlike their optical analogies. In particular, in the case of magnetic films, the focusing into one spatial point takes place in a stationary regime [16]. This is explained by the fact that longitudinal dispersion can not be neglected in magnetic films. In the present paper, it is shown that even that gap could be filled by considering in-plane magnetized films where the carrier wave vector is neither parallel nor perpendicular to the static magnetic field direction. The conditions for stationary and stable self-focusing magnetostatic beams in wide magnetic films are found.

Although the linearized spin-wave solutions are well known for arbitrary directions between the wave vector and magnetizing field [18,19], the nonlinear situation has been studied only for the cases when carrier wave vector is either parallel or perpendicular to the magnetizing field direction [1–11,14–16] (one exception is Ref. [20] where magnetized field is tilted from the film normal in order to control the nonlinear coefficient, but this study does not pertain to the present consideration). Only very recently have the nonlinear effects for the general case been investigated [21], and soliton solutions have been found for angles (between wave vector and magnetic field) other than 0 or 90 degrees. However, such solutions are stable only in the quasi-one-dimensional case and they become unstable when considering wide samples. As it will be shown below in the two dimensional case, only self-focusing beam solutions are stable. Note that bullet like solutions are metastable and they decay after either edge reflection or interaction with each other [14,15], (metastability takes place due to the compensation of instability by dissipation).

It should be especially mentioned that the wave processes are easily accessible from the surface by a variety of the methods, such as inductive probes [22], thermo-optical methods [23] and a recently developed method of space and time resolved Brillouin light scattering [24]. Therefore it does not seem problematic to detect the nonlinear localizations predicted in this paper.

2 Problem geometry and reduction to 1D NLS

For arbitrary directions of the carrier wave vector $\vec{k}$ with respect to a static magnetic field $\vec{H}$, the problem for nonlinear wave processes (magnetostatic and Landau-Lifshitz equations) in in-plane magnetized thin film reduces to the

[a] e-mail: khomeriki@hotmail.com

following nonlinear equation for the wave envelope u (see e.g. Ref. [21]):

$$i\left(\frac{\partial u}{\partial t}+v_y\frac{\partial u}{\partial y}+v_z\frac{\partial u}{\partial z}\right)+\frac{\omega''_{yy}}{2}\frac{\partial^2 u}{\partial y^2}+\frac{\omega''_{zz}}{2}\frac{\partial^2 u}{\partial z^2}+\omega''_{yz}\frac{\partial^2 u}{\partial y\partial z}-N|u|^2u=-i\omega_r u, \quad (1)$$

where $\vec{r}$ stands for the radius vector lying in the sample plane yz, x is a coordinate along the direction perpendicular to the film, and z is a direction of static magnetic field; v_y and v_z are the components of the group velocity and they could be calculated from the linearized dispersion relation (but see below) $\vec{v}\equiv\partial\omega/\partial\vec{k}$; ω and $\vec{k}$ are carrier frequency and wave vector of the magnetostatic spin wave; $\omega''_{\beta\gamma}\equiv\partial^2\omega/\partial k_\beta\partial k_\gamma$ (indexes β and γ take the values y and z). u is a complex envelope of the relative magnetization vector $\vec{m}=\vec{M}/M_0$ (M_0 is a static magnetization along z coinciding with the direction of magnetizing field):

$$m_x=u\,\frac{ik_x\omega_H-k_y\omega_0}{k\sqrt{\omega_H^2+\omega_0^2}}\;e^{i(\omega t-\vec{k}\vec{r})};$$

$$m_y=u\,\frac{k_x\omega_0+ik_y\omega_H}{k\sqrt{\omega_H^2+\omega_0^2}}\;e^{i(\omega t-\vec{k}\vec{r})}. \quad (2)$$

Here $\omega_H=gH$, $\omega_M=4\pi gM_0$ and $\omega_0\equiv\omega(k=0)=\sqrt{\omega_H(\omega_H+\omega_M)}$ (g is the gyromagnetic ratio for electrons). The above expressions (2) have been derived [21] in the limit $kd\ll 1$ (d is a film thickness), and this condition will be further used in this paper in order to simplify the analytic calculations. Then in this case, the dispersion relation could be expressed as an expansion over the small parameter kd, and keeping only the terms up to second order of this parameter, the following expression is obtained [21]:

$$\omega=\omega_0+\frac{\omega_M}{4\omega_0}\frac{d}{k}\left(\omega_M k_y^2-\omega_H k_z^2\right)-\frac{\omega_M^2}{32\omega_0^3}\frac{d^2}{k^2}\left(\omega_M k_y^2-\omega_H k_z^2\right)^2+\frac{\omega_M}{4\omega_0}d^2\left(\omega_H\frac{k_z^2}{3}-\omega_M k_y^2\right). \quad (3)$$

The phenomenological dissipation parameter ω_r in (1) is weak, but it plays an important role in the stabilization of the localized solutions (see e.g. Ref. [16]) and finally, the nonlinear coefficient N in the case of in-plane magnetized films is always negative and reads as follows [1] $N=-\omega_H\omega_M/4\omega_0$.

In order to eliminate the nondiagonal term with coefficient ω''_{yz} in equation (1), a new frame of references $\eta\xi$ should be introduced (see Fig. 1a). Let us rotate the frame of references yz by the angle ϑ defined from the following relation: $\tan(2\vartheta)=2\omega''_{yz}/(\omega''_{zz}-\omega''_{yy})$. Then from (1) (2+1) dimensional (two spatial and one temporal dimensions)

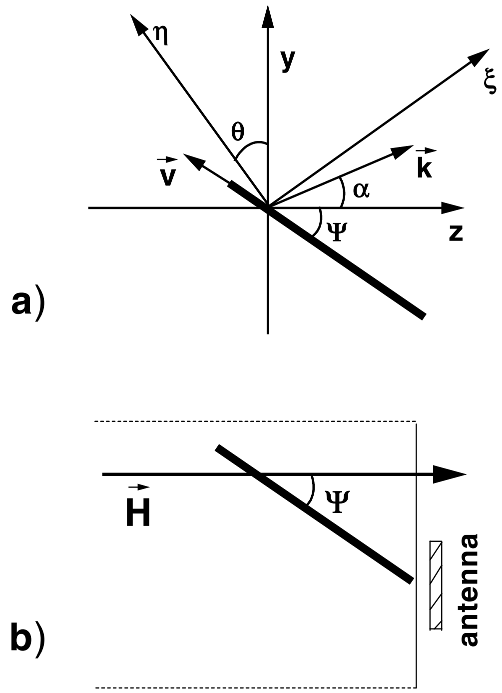

Fig. 1. Geometry of the problem in the case of a stationary self-focusing beam. a) Orientations of the wave vector $\vec{k}$, static magnetic field $\vec{H}$, group velocity $\vec{v}$ and diagonalizing frame of references $\eta\xi$. Thin line indicates the direction of the beam. b) The possible experimental setup for observation of the stable magnetostatic self-focusing beam. Ψ indicates an angle between static magnetizing field and preferential radiation direction of the antenna (see also Figs. 5a and 6b in Ref. [16]).

nonlinear Schrödinger (NLS) equation is obtained:

$$i\left(\frac{\partial u}{\partial t}+v_\xi\frac{\partial u}{\partial \xi}+v_\eta\frac{\partial u}{\partial \eta}\right)+\frac{R}{2}\frac{\partial^2 u}{\partial \xi^2}+\frac{S}{2}\frac{\partial^2 u}{\partial \eta^2}-N|u|^2u=-i\omega_r u, \quad (4)$$

which reduces to the standard form (without first spatial derivatives) after introducing the following coordinate transform (moving frame) $\eta\to\eta-v_\eta t$ and $\xi\to\xi-v_\xi t$. In equation (4) coefficients R and S are dispersion and diffraction coefficients, respectively, explicit form of which are given in reference [21], and v_η and v_ξ are the group velocity components with respect to the new reference frame $\eta\xi$. Usually in isotropic systems, the transvers component of the group velocity v_η is equal to zero (the same happens in the case of nonlinear magnetostatic waves when the carrier wave vector is either parallel or perpendicular to a static magnetic field [1,2]). But in general for anisotropic systems, v_η is not equal to zero.

As is well known, the (2+1) NLS equation does not permit [25,26] stable localized solutions, irrespective of the relative sign of the coefficients S, R and N. Only the metastable bullet like localizations appear [12–15], due to the compensation of wave instability by the dissipation.

However, in restricted geometries (waveguides), transverse instabilities do not develop and the diffraction term could be neglected thus allowing reduction to a (1+1) NLS equation. Such geometries have been used in order to observe solitons in optical fibers and magnetic film waveguides.

Another possibility to see localized solutions is the absence of the dispersion term $R = 0$. Such a situation is realized in nonlinear optics where dispersion is negligible in comparison with diffraction. In this case, a spatial soliton solution (self-focusing beam) is stable [17]. However, in the case of magnetic films, the dispersion coefficient could not be neglected and the beam like solutions have not been observed under the experimental conditions considered until now. In the present paper, an experimental setup is suggested where the angle α between carrier wave vector and magnetizing field takes the value for which the dispersion coefficient is nearly zero ($R \simeq 0$). As far as all the coefficients of the (2+1) NLS equation (4) are defined from the dispersion relation (3), they could be expressed as functions of wave number k and the angle α. Particularly, in the limit of small k ($kd \to 0$) the dispersion coefficient R does not depend on k. Therefore, the problem is reduced to finding such as α which makes R equal to zero for a given static field and sample parameters. As numerical simulations show, for each magnitude of the static magnetic field, it is possible to find such an angle.

Actually it is not necessary to have such α-s for which R is exactly zero. One can neglect the role of dispersion in formation of nonlinear waves if the following inequality holds $R \ll \omega_r/k^2$, i.e. the wave dissipates faster than dispersion effects take place. Similarly, one can neglect higher order dispersion terms in comparison with dissipation, as long as they are proportional to the factor $(kd)^3$ ($kd \ll 1$ in this paper). At the same time, the diffraction term in my calculations is much larger than the dissipation one. Thus, only the diffraction and nonlinearity determine the dynamics of nonlinear wave, and reduction to the (1+1) NLS equation is justified.

3 Stationary self focusing beam solution

Considering a standard stationary situation $\partial/\partial t = 0$, and using coordinate transformation $\eta \to \eta - (v_\eta/v_\xi)\xi$ in the case of close to zero dispersion ($R \simeq 0$), equation (4) reduces to the (1+1) NLS equation (ξ plays the role of time) with a stable spatial soliton solutions. For instance, one soliton solution could be presented analytically as follows:

$$|u| = |u|_{max}\,\mathrm{sech}\left\{\frac{\eta - (v_\eta/v_\xi)\xi}{\Lambda}\right\} \quad (5)$$

corresponding to the self-focusing beam along the direction of the group velocity $\vec{v}$. Here the beam width Λ is defined as follows:

$$\Lambda = \left|\frac{S}{N}\right|^{1/2}\frac{1}{|u|_{max}}. \quad (6)$$

Note that the amplitude $|u|_{max}$ decays and the beam width increases with distance, thus taking into account the weak dissipation effects.

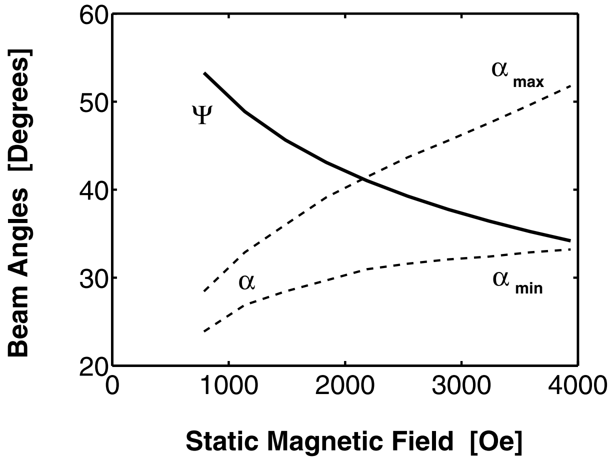

Fig. 2. Range of angles for which the self-focusing beam regime could be realized. α (dashed borders) stands for the angle range between carrier wavevector and static magnetic field, while Ψ (solid line) is a corresponding angle between the direction of the group velocity and the static magnetic field.

Knowing angles α for which the dispersion effect can be neglected, it is easy to calculate the diffraction coefficient S and the group velocity $\vec{v}$. Then, the angle Ψ between $\vec{v}$ and the z-axis for that particular value of angle α can be found. The observation of stationary self-focusing is possible only for the mentioned radiation direction. In Figure 2 the dependences of angles α and Ψ versus static magnetic field are presented.

4 Discussion of possible experimental set-up

The following values for the film parameters are used in the calculations: film thickness $d = 5$ μm; demagnetizing field $H_M = \omega_M/g = 1750$ Oe, and the dissipation parameter is taken to be $\omega_r = 5 \times 10^6$ s^{-1} as in reference [16]. As discussed above, the beam self-focusing process is realized if dispersion effects can be neglected, i.e. if the following inequality is satisfied $R \ll \omega_r/k^2$. The calculations are made for a carrier wave number of $k = 50$ cm^{-1}. In Figure 2 the range of angles between $\vec{k}$ and the static magnetic field for which the above inequality holds is presented (dispersion effects can be neglected) and, besides that, the range of angles α corresponds to a single direction of the group velocity. As seen for the values of the static magnetic field $H_0 > 2500$ Oe almost the same direction of group velocity corresponds to the wide range of validity of beam generation regime. That direction must coincide with a preferential direction of the magnetostatic wave radiation from short antenna or point like source. The ways to experimentally vary the preferential direction of radiation in a linear regime has been suggested in reference [16], and here we suggest the use of the same method in a nonlinear regime in order to observe self-focusing beams. Thus the antenna should be oriented in such a way that the angle between its preferential direction of radiation and the static magnetic field coincides

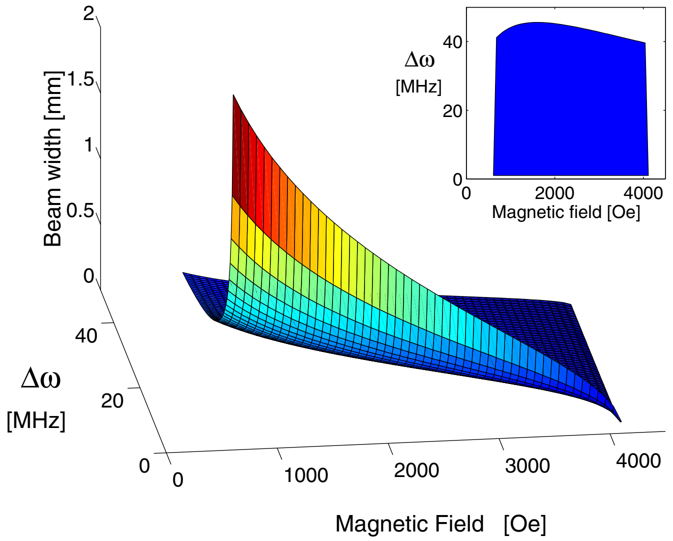

Fig. 3. Self focusing beam width versus detuning of the carrier frequency $\Delta\omega = \omega_0 - \omega$ and static magnetic field. Inset shows the projection of the plot on the horizontal plane where the filled area indicates considered range. The static magnetic field is restricted by the boundaries $0.3\omega_M < \omega_H < 2.37\omega_M$ (see the text), while the upper limit corresponds to the condition $kd < 0.1$. The following parameters are used for the calculations: relative amplitude of the beam $|u|_{max} = 0.1$; film thickness $d = 5$ μm and demagnetizing field $H_M = 1750$ Oe.

with the derived angles for the beam group velocity (angles Ψ in Figs. 1 and 2). It should be mentioned that the calculations show (in the considered limit $kd \to 0$) that the required angles between radiation direction and static magnetic field do not depend on $|k|$, nor consequently on the carrier frequency ω of the excitation (note that this happens when the dispersion coefficient R is negligible). However, the diffraction coefficient S and, as a result, the beams width are inversely proportional to kd.

In Figure 3, a three dimensional plot of the beam width Λ versus detuning of the carrier frequency $\Delta\omega = \omega_0 - \omega$ and magnitude of the static magnetic field is presented. The magnetic field varies within the boundaries $0.3\omega_M < \omega_H < 2.37\omega_M$ where the lower boundary appears from the requirement that a three magnon processes should not take place [1] (otherwise the localizations will decay rapidly), while above the upper limit, the diffraction coefficient S becomes negative and consequently (according to the Lighthill criterion [27]) the self-focusing process does not take place.

5 Conclusions

In conclusion, the conditions for the observation of stationary self-focusing beams in magnetic films are found. It is suggested that such localizations could be observed along preferential direction of the antenna's radiation.

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